SageManifolds: differential geometry with SageMath

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SageMath in a few words

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SageMath is based on Python

- no need to learn any specific syntax to use it
- Python is a powerful *object oriented language*, with a neat syntax
- SageMath benefits from the Python ecosystem (e.g. Jupyter, NumPy, Matplotlib)

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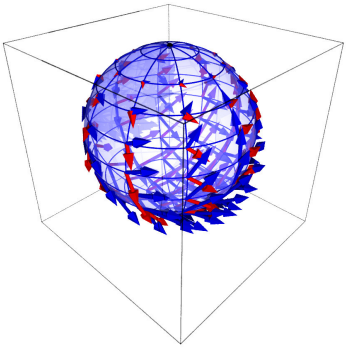
SageMath is developed by an enthusiastic community

- mostly composed of mathematicians
- welcoming newcomers

Differential geometry with SageMath

SageManifolds: extends SageMath towards **differential geometry** and **tensor calculus**



Vector frame of stereographic coordinates on $\mathbb{S}^2$

- https://sagemanifolds.obspm.fr
- $\sim$ 119,000 lines of Python code
- fully included in SageMath (after review process)
- $\sim$ 30 contributors (developers and reviewers), cf. https://sagemanifolds.obspm.fr/authors.html
- dedicated mailing list
- help desk: https://ask.sagemath.org

Everybody is welcome to contribute

$\Longrightarrow$ visit https://sagemanifolds.obspm.fr/contrib.html

Current status

Already present (SageMath 10.10):

- differentiable manifolds: tangent spaces, vector frames, tensor fields, curves, pullback and pushforward operators, submanifolds
- vector bundles (tangent bundle, tensor bundles)
- standard tensor calculus (tensor product, contraction, symmetrization, etc.), even on non-parallelizable manifolds, and with all monoterm tensor symmetries taken into account
- Lie derivative along a vector field
- differential forms: exterior and interior products, exterior derivative, Hodge duality
- multivector fields: exterior and interior products, Schouten-Nijenhuis bracket
- affine connections (curvature, torsion)
- pseudo-Riemannian metrics
- computation of geodesics by numerical integration

Current status

Already present *(cont'd)*:

- some plotting capabilities (charts, points, curves, vector fields)
- parallelization (on tensor components) of CPU demanding computations
- extrinsic geometry of pseudo-Riemannian submanifolds
- series expansions of tensor fields
- symplectic manifolds
- 2 symbolic backends: Pynac/Maxima (SageMath's default) and SymPy

Future prospects:

- more symbolic backends (Giac, FriCAS, ...)
- more graphical outputs
- spinors, integrals on submanifolds, variational calculus, etc.
- connection with numerical relativity: use SageMath to explore numerically-generated spacetimes

The SageManifolds project in a nutshell

Symbolic calculus on manifolds in the free Python-based system SageMath

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Many examples available at https://sagemanifolds.obspm.fr/examples.html

Want to join the project or to simply stay tuned?

visit https://sagemanifolds.obspm.fr/
(download, documentation, example notebooks, mailing list)

It's time for the hands-on session!

Visit https://sagemanifolds.obspm.fr/handson2026/